RADIATOR: program for simple transition radiation calculation.

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1 Introduction

Since its prediction [10] and experimental discovery, the Transition Radiation (TR) was thoroughly investigated and became a usual technique in modern physics experiments. In most of the cases TR detectors are part of a complicate geometry setup, which provides not only the particle identification capability, but the track finding and measurement tools in a busy environment as well.

Therefore simulations of the transition radiation has to be an integral part of a more general detector simulation tools, with

- a description of an arbitrary complex geometry,
- detailed description of the signal registration process, and
- a reasonable, or at least controllable consumption of the computing resources.

TR simulation is currently integrated in the large simulation framework such as GEANT [1]. Despite all the advantages available in GEANT, the configuration (mainly materials and size) of radiator and detector combination to produce and register TR component have to be developed first. Thus, to provide a fast tool to calculate TR yield produced by the particle and registered in the detector has been developed. This tool has been previously used in the development of Transition Radiation Tracker (TRT) successfully working [3] in the ATLAS experiment at the LHC [4]. The name of the tool is "RADIATOR" and it is available online: http://radiator.hepforge.org/.

2 Calculation algorithm

We are considering the TR yield as described in papers [8, 12, 9], starting with the case of weak absorption, regular foil radiator and flat geometry. The following text is describing how to calculate TR yield for a simple case with flat geometry, combining standard techniques to describe TR absorption and other physics processes in a common way previously described in [11].

2.1 Theoretical predictions

It is well known [3] that TR emittance from a single surface can be described as

$$\frac{d^2S_0}{d\theta d\omega} = \frac{2\alpha\hbar\theta^3}{\pi} \times \Big(\frac{1}{\gamma^{-2}+\theta^2+\omega_1^2/\omega^2} - \frac{1}{\gamma^{-2}+\theta^2+\omega_2^2/\omega^2}\Big), \tag{1}$$

where γ is the particle Lorentz-factor and $\omega_{1,2}$ are the plasma frequencies of the foil and gap materials, respectively (common example is $\omega_{air} = 0.7\,\mathrm{eV}$ and $\omega_{mylar} = 21\,\mathrm{eV}$).

In practice, Equation 1 is of a little interest for the detector simulation purpose, as it requires from a simulation program an ability to propagate not just photons but their amplitudes and to properly take into account the interference after the propagation step.

Radiation from a stack of N regular foils of thickness l_1 separated by distances l_2; can be described by the following expression [8]:

$$\frac{d^2S}{d\theta d\omega} = \frac{d^2S_0}{d\theta d\omega} 4\sin^2\Big(\frac{l_1}{Z_1}\Big) \times \frac{\sin^2[N(l_1/Z_1+l_2/Z_2)]}{\sin^2(l_1/Z_1+l_2/Z_2)}. \tag{2}$$

This equation takes interference effects into account using a notion of so-called *formation zones* $Z_{1,2}$, where

$Z_{1,2} = \frac{2c}{\omega}(\gamma^{-2}+\theta^2+\omega_{1,2}^2/\omega^2)^{-1}$,

are distances along the particle trajectory inside and between the radiators, after which a separation between the particle and emitted photons becomes of the order of the photon wavelength.

Equation 2 describes the photon density at distances significantly larger than the formation zone with the interference taken into account. However, practically it is still not perfect and certainly creates at least a numeric accuracy problem for the implementation. In fact, its numerical integration over TR emission angles is difficult and may be very time consuming due to presence of multiple sharp peaks in the angular distribution produced by the photon amplitude interference.

An approximate analytical integration of Eq. 2 over TR emission angles has been done in Refs. [12, 9] by replacing radiation angular peaks with delta-functions. This leads to the following expression for the energy, emitted at frequency ω:

$$\frac{dS}{d\omega} = 2\alpha\hbar c N \Big[\frac{(\omega_1^2-\omega_2^2)(l_1+l_2)}{\omega}\Big]^2 \times \sum_{r=r_{min}}^{r=r_{max}} \frac{(r-R_m)\sin^2\big(\frac{\pi(r-R_1)l_2}{l_1+l_2}\big)}{(r-R_1)^2(r+R_2)^2}, \tag{3}$$

where

$R_{1,2} = \frac{e}{4\pi\hbar c}\frac{\omega_1^2-\omega_2^2}{\omega} l_{1,2}$,
$R_m = \frac{e}{4\pi\hbar c}\Big[\frac{w(l_1+l_2)}{\gamma^2} + \frac{l_1\omega_1^2+l_2\omega_2^2}{\omega}\Big]$,
$r_{max} = \frac{e}{2\pi\hbar c}\sqrt{(l_1\omega_1^2+l_2\omega_2^2)(l_1+l_2)}$,
$r_{min} = \min(R_m, \frac{r_{max}}{\gamma})$.

Several authors (e.g. Refs. [5, 7]), who verified the validity of this analytical integration, claim it to be accurate within at least 10%. Our experience shows that it is even higher as soon as the conditions $Z \gg \pi Z_2$ is satisfied, i.e. photons are absorbed at a distance from radiators significantly larger that the formation zone of the air - a condition which is satisfied practically in all cases. A difference between the results of the numeric integration of Eq. 2 and the calculations of Eq. 3 goes below 1% with sufficient accuracy of the numeric integration of Eq. 2.

It is interesting to mention that another expression often used in Ref. [7] for the number of TR photon integrated over emission angles:

$$\frac{dN}{d\omega} = \frac{4\alpha N}{\omega(1+\tau)} \sum_n \theta_n \Big(\frac{1}{\rho_1+\theta_n} - \frac{1}{\rho_2+\theta_n}\Big)^2 \times (1-\cos(\rho_1+\theta_n)), \tag{4}$$

where

$\rho_i = \frac{\omega l_1}{2c}\Big(\gamma^{-2} + \frac{\omega_i^2}{\omega^2}\Big)$,
$\theta_n = \frac{2\pi n - (\rho_1+\tau\rho_2)}{1+\tau}$,
$\tau = l_2/l_1$.

It gives the same results as Eq. 3 and is in fact equivalent to it. Thus, Eq. 4 is used to calculate TR yield from a particle.

2.2 Basic proprieties of the transition radiation

The basic TR properties important for the detector applications can be summarised as following:

- Radiation peaks in the forward direction at $\theta \approx \sqrt{\gamma^{-2}+\omega_2^2} \approx 10^{-3}$ or smaller.

- The bulk of the energy is emitted at $\omega \approx \gamma\omega_2$, i.e. in the most energetic part of the spectra.
- Above the cut-off energy $\sim \gamma\omega_1$ nothing is emitted.
- When the distance between foils falls below the formation zone, (un)coherence significantly affects the emitted radiation spectra.

2.3 Absorption effects

Although formulae 3 and 4 describe the TR yield at distances much larger than formation zone, they can be interpreted as a local TR production density per unit length as they are proportional to the number of foils crossed.

In this assumption the absorption can be taken into account as a convolution of a continuous TR production and a local absorption so that

$$S_{withabsorption} = \int_0^L dx \frac{dS}{dx} \mathrm{e}^{-\sigma(L-x)} = S_N \frac{1 - \mathrm{e}^{-N(\sigma_1 l_1 + \sigma_2 l_2)}}{N(\sigma_1 l_1 + \sigma_2 l_2)}. \quad (5)$$

Another equivalent way of taking absorption into account is the introduction of the number of equivalent radiators:

$$N_{eff} = \frac{1 - \mathrm{e}^{-N(\sigma_1 l_1 + \sigma_2 l_2)}}{1 - \mathrm{e}^{-(\sigma_1 l_1 + \sigma_2 l_2)}}. \quad (6)$$

2.4 Crucial experimental tests

A number of experiments have been done in the beginning of 1970s to verify the accuracy of theory prediction. Two of them [7, 6] are especially important and can be considered as crucial tests, as they measured purely the contribution of the TR as a function of the photon energy. Magnetic field was used to separate electrons from the TR in order to avoid the background of the ionisation energy loss. In both cases the TR photons where measured by a solid-state photon detector with a well-known sensitivity over a large range of photon energies, thus providing not only the integral, but the full-energy dependence of the TR yield.

Both experiments have shown a good coincidence between the theory and measured data. In particular, the interference effects were confirmed with good accuracy both in case of thin ($l_2 \ll Z_2$) and thick ($l_2 \sim Z_2$) radiators.

The absolute yield of the TR measured in these experiments in some cases is less than the theory prediction, but this occasional discrepancy can be apparently explained by systematic errors like partial angular coverage of the detector setup or not perfect detection efficiency.

Nevertheless, the shape of the measured spectra coincides with the one predicted by formulae 3 and 4 well. The most striking confirmation of the theoretical predictions is the precise coincidence in the position of the radiation minima and maxima, thus confirming the accuracy of the description of the interference mechanism based in the introduction of formation zones.

3 Users' guide

The RADIATOR code works within the ROOT framework (ROOT 5.34 or higher [2]). The code is contained in the macro Radiator_vX.Y.C. The version described here is 2.0. While the functionality is quiet wide for calculation of TR yield, users are encouraged to write their own functions or to modify the code in order to access results of TR calculation for required configuration.

The macro requires a .txt file as an input, containing a description of the configuration for the calculation. Below is provided the listing of the file, which provide description of the system consisting of two radiator-detector structures :

```
TR yield calculation - all data are in mkm (1e-3 mm)
+-----+-----+-----+-----+L rad+Nfoil+Dfoil+-----+-----+-----+-----+-----+
 SET  :BAREL:    1*1.   :    1*1.       0*0.        0*0.        0*0.
 BLOCK:NORMAL    1.    1.    1.           :
 ELEM :RADIA:PPROP+AIR  :158e3.  150*62   :
      :WALL :MYLAR      :    0.05         :
      :CHAMB:XE         :  3e4.           :
      :WALL :MYLAR      :    0.05         :
 ELEM :RADIA:PPROP+AIR  :158e3.  150*62   :
      :WALL :MYLAR      :    0.05         :
      :CHAMB:XE         :  3e4.           :
      :WALL :MYLAR      :    0.05         :
 THRES:     :  3.0   3.5   4.0   4.5   5.0   5.5   6.0   6.5
 ENERG:     :  0.5   500  .005
 GAMMA:     :   -1
+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
 VAR  : G-1 :  5e2|  1e3|  3e3|  6e3| 8e3 |  1e4|  2e4|  4e4|     |     |
 END  : OF EXAMPLE
```

Here the strings correspond to:

- 1st line describes the geometry of the detector, here it is "BAREL", and ordering of the structure, where radiator-detector equals to "1*1.";
- 2nd line describes the structure of the detector materials, here it is thin mylar wall, detecting volume and mylar wall again, which is "1.   1.   1.";

- 3rd line describes the materials and structure of the radiator structure, which is polypropylene-air structure (“PPROP+AIR”) 158 mm long and consists of 150 foils of 62 μm thickness;
- Lines from 4 to 6 describe the materials of the detector, where it is two mylar walls of 50 μm and xenon as the main detecting volume of 3 sm;
- Lines from 7 to 10 describe the materials and structure of the next radiator-detector set;
- Lines 11 and 12 are used to set energy thresholds to make first calculations of the detected yield and set the binning of the histograms, respectively;
- Line 15 lists the gamma-factor values used to calculate the yield of TR photons.

3.1 Running the Code

To calculate TR yield for 8 values of gamma-factor using input file described above one has to construct a Radiator object using “root” infrastructure:

```
root[0] gROOT->ProcessLine(".L Radiator_vX.Y.C+");
root[1] radiator("InputFileName.txt", "OutputFileName.root");
```

where the first line compiles the code and the second loads the setup for the calculations.

3.2 Provided functionality

The program is designed to calculate TR yield for variety of the radiator and detector materials. The list of materials and abbreviation which can be used to setup the configuration is provided in the Table 1.

With respect to the configuration the program is providing several TR yields:

- Generated TR photon yield;
- The yield after absorption in the radiator materials, which is based on NIST data base [13];
- Incident yield after absorption in the detector materials;
- Registered in the detector TR photon yield, taking into account interaction with electrons on the L and K shells.

Figure 1 is showing the output of RADIATOR program for 150 foils of 62 μm thick mylar radiator, with TR yield registered by 3 sm xenon detector.

For detector development purpose, two last maxima of the yield are used to measure the contribution of the TR yield i the particle detection. Figure 2 is showing the comparison of the Generated Incident and registered spectra obtained with RADIATOR program for 150 foils of 62 μm thick mylar radiator, with TR yield registered by 3 sm xenon detector.

Material	Abbreviation
Hydrogen	H
Helium	HE
Carbon	C
Nitrogen	N
Nitrogen	N
Oxigen	0
Xenon	XE
Berilium	BE
Litium	LI
Boron	B
Bismuth	BIS
Germanium	GE
Polistirol	PST
Polypropylene	PPR
Mylar	MYL
Air	AIR
CH_2+20%B	20B
CH_2+30%B	30B
BGO	BGO
LIT	LIT

Table 1: The list of implemented materials and corresponding abbreviations.

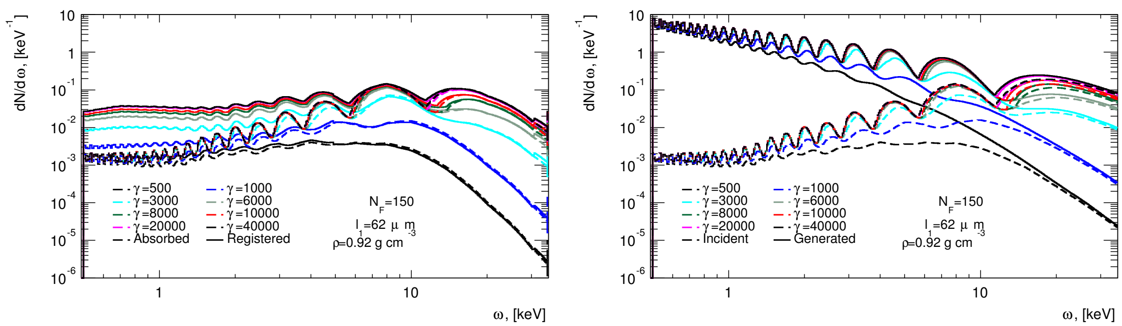

Figure 1: TR photon spectra, calculated for eight gamma-factor values with RADIATOR program for 150 foils of 62 μm thick mylar radiator, with TR yield registered by 3 sm xenon detector.

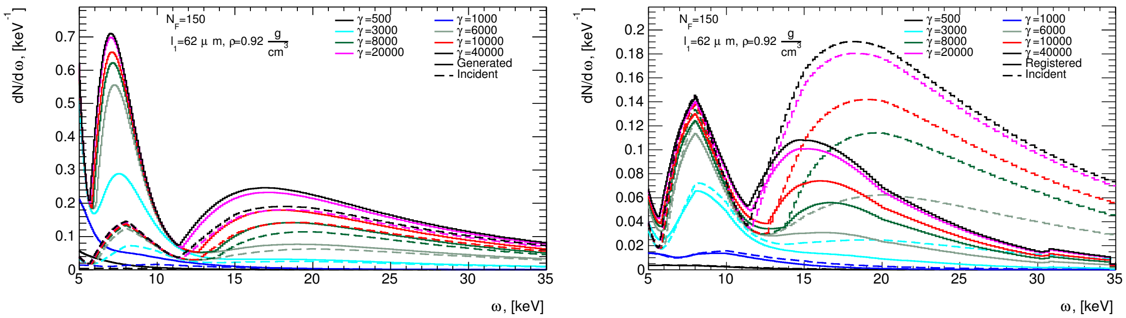

Figure 2: TR photon spectra, calculated for eight gamma-factor values with RADIATOR program for 150 foils of 62 μm thick mylar radiator, with TR yield registered by 3 sm xenon detector.

4 Summary

Transition radiation is a useful tool for particle identification, used by variety of experiments. This document describes a tool, which is using common TR emission calculations to provide a flexible tool for initial calculations aimed for development of radiator-detector configuration in future experiments.

The code is accessible online and can be used within user code or in a standalone mode allowing analysis of various distributions. The authors welcome comments on the code and suggestions on how to make it more useful to both experimentalists and theorists.

References

[1] J. Allison et al., *Recent developments in Geant4*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **835** (2016) p. 186 , ISSN: 0168-9002, URL: `http://www.sciencedirect.com/science/article/pii/S0168900216306957`.

[2] I. Antcheva et al., *ROOT A C++ framework for petabyte data storage, statistical analysis and visualization*, Computer Physics Communications **180** (2009) p. 2499 , URL: `http://www.sciencedirect.com/science/article/pii/S0010465509002550`.

[3] ATLAS Collaboration, "Basic ATLAS TRT performance studies of Run 1", tech. rep. ATL-INDET-PUB-2014-001, CERN, 2014, URL: `http://cds.cern.ch/record/1669603`.

[4] ATLAS Collaboration, *The ATLAS Experiment at the CERN Large Hadron Collider*, JINST **3** (2008) S08003, URL: `http://dx.doi.org/10.1088/1748-0221/3/08/S08003`.

[5] M. L. Cherry, *Measurements of the Spectrum and Energy Dependence of x-Ray Transition Radiation*, Phys. Rev. **D17** (1978) p. 2245.

[6] M.L. Cherry and et al., Phys. Rev. D **10** (1974) p. 3594.

[7] C.W. Fabjan and W. Struczinski, Phys. Rev. Lett. B **57** (1975) p. 483.

[8] G.M. Garibian, Zh. Eksp. Teor. Fiz. **39** (1960) p. 332,
(Sov.Phys.- JETF 12 (1961) 237).

[9] G.M. Garibian, Zh. Eksp. Teor. Fiz. **60** (1971) p. 39,
(Sov.Phys.- JETF 33 (1971) 23).

[10] V. L. Ginzburg and I. M. Frank, *Theory of transition radiation*, Zh. Eksp. Teor. Fiz. **16** (1946) p. 15,
Ginzburg V L, Frank I M 1945 J. Phys. USSR 9 353 (brief English language version).

[11] Pavel Nevski, *Advances in the simulation of Transition Radiation Detectors*, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **522** (2004) p. 116 , {TRDs} for the Third Millenium. Proceedings of the 2nd Workshop on Advanced Transition Radiation Detectors for Accelerator and Space Applications, ISSN: 0168-9002, URL: http://www.sciencedirect.com/science/article/pii/S016890020400107X.

[12] M.L. Ter-Mikaelian, Nucl. Phys. **24** (1961) p. 43.

[13] *XCOM: Photon Cross Sections Database*, NIST, PML, Radiation Physics Division, URL: https://www.nist.gov/pml/xcom-photon-cross-sections-database.